



Reg. No. :

Name :

**Fifth Semester B.Tech. Degree Examination, October 2016
(2013 Scheme)**

**13.502 : ENGINEERING MATHEMATICS – V (FR)
(Advanced Mathematics and Queueing Models)**

Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks :

1. Define (i) Basic solution (ii) Basic feasible solution (iii) Degenerate solution (iv) Optimal solution of an LPP.

2. Maximize $z = x_1 + x_2$, subject to $x_1 + x_2 \leq 2$, $x_1 \geq 1$, $x_2 \leq 1$, $x_1, x_2 \geq 0$.

3. Find the dual of the LPP

$$\text{Minimize } z = 4x_1 + 2x_2 + x_3$$

$$\text{Subject to } x_1 + x_2 \leq 10$$

$$3x_1 + x_2 + x_3 \geq 23$$

$$7x_1 - x_3 = 6$$

$$x_1, x_2, x_3 \geq 0.$$

4. Explain the following :

i) Free float

ii) Independent float.

5. A TV repairman repair the sets in the order in which they arrive and expects that the time required to repair a set has an exponential distribution with mean 30 minutes. The sets arrive in a Poisson fashion at an average rate of 10 per 8 hours a day.

a) What is the expected idle time per day for the repairman ?

b) How many TV sets will be there awaiting for the repair ?





PART - B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

Module - I

6. A refinery uses two different crude oils, light crude oil and heavy crude oil to produce gasoline, heating oil, jet fuel and lube oil. The proportion of the two types of crude oils used for the production of these products are given in the table below.

	Gasoline	Heating Oil	Jet fuel	Lube Oil
Light Crude Oil	0.4	0.2	0.3	0.1
Heavy Crude Oil	0.3	0.45	0.1	0.05

The demand is 8 million barrels of gasoline, 6 million barrels of heating oil, 7 million barrels of jet fuel and 3 million barrels of lube oil. If light crude oil costs Rs. 60 per barrel and heavy crude oil costs Rs. 50 per barrel, determine graphically the amounts of the two types of oils to be purchased so that the demands are fulfilled and that the total cost is minimum.

7. Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$
subject to

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 15 \\ 2x_1 + x_2 + 5x_3 &= 20 \\ x_1 + 2x_2 + x_3 + x_4 &= 10 \\ x_1, x_2, x_3, x_4 &\geq 0. \end{aligned}$$

Module - II

8. Apply the principle of duality to solve the LPP

$$\text{Minimize } z = 2x_1 + x_2$$

subject to

$$\begin{aligned} 3x_1 + x_2 &\geq 3 \\ 4x_1 + 3x_2 &\geq 6 \\ x_1 + 2x_2 &\geq 3 \\ x_1, x_2 &\geq 0. \end{aligned}$$

9. a) The owner of a small machine shop has four machines available to assign the jobs. Five jobs are offered with expected profit in Rs. for each machine on each job are given in table below. Determine the assignment of machines to jobs that will result in a maximum profit. Which job should be declined?

		Jobs				
		A	B	C	D	E
Machines	M ₁	62	78	50	101	82
	M ₂	71	84	61	73	59
	M ₃	87	92	111	71	81
	M ₄	48	64	87	77	80



- b) A salesman has to visit five cities A, B, C, D and E. The distances between these cities are given below. If the salesman starts from the city A and has to come back to city A, which route should he select so that the total distance travelled by him is minimized.

	A	B	C	D	E
A	-	4	7	3	4
B	4	-	6	3	4
C	7	6	-	7	5
D	3	3	7	-	7
E	4	4	5	7	-

Module - III

10. a) What is meant by *critical path* ? What is its importance in project scheduling ?
 b) A project consists of a series of tasks labelled A, B, ... , H, I with the following relationships.

A < D, E; B, D < F; C < G; B < H; F, G < I.

(W < X, Y means that X and Y cannot start until W is completed and X, Y < W means W cannot start until both X and Y are completed) Construct the network diagram. Find also the minimum time for completion of the project, when the time (in days) of completion of each task is as follows :

Task	A	B	C	D	E	F	G	H	I
Time	8	10	8	10	16	17	18	14	9

11. A project is composed of 7 activities whose completion time estimates are listed in the following table.

Activity	Duration (in weeks)		
	Optimistic	Most likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15



- Draw the project network.
- Find the expected duration and variance for each activity.
- What is the expected project length ? Calculate its standard deviation.
- Find the probability that the project will be completed at least 4 weeks earlier than the expected time.
- What is the probability that the project will not be completed in 19 weeks ?



Module - IV

12. a) The arrivals of telephone calls at a telephone booth are according to a Poisson process with an average time of 12 minutes between arrival of two consecutive calls. The length of the telephone call is assumed to be exponentially distributed with mean 4 minutes.
- Find the average queue length.
 - Determine the probability that the person arriving at the booth will have to wait.
 - What is the probability that an arriving customer has to wait for more than 15 minutes before the phone is free ?
 - The telephone company will install a second booth if a customer is expected to wait at least 5 minutes for the phone. Find the increase in arrival rate which will justify the second booth.
- b) The arrival of cars to a single server service centre follows Poisson distribution with mean 18 per hour. Time taken for the service a car is exponentially distributed with mean 10 minutes. If the service centre can accommodate at most three cars including the one being served, find
- Probability that an arriving customer need not wait.
 - Expected number of cars in the service centre.
 - Expected number of cars waiting for service.
 - Expected time a customer spends in the service centre.
 - Expected time a customer spends in the queue.
13. a) A road transport company has one reservation counter serving the customers. The customers arrive in a Poisson fashion at the rate of 4 per hour. The service time for each customer is exponentially distributed with mean 10 minutes. Find the
- Probability that a customer has to wait for service.
 - Average number of customers in the queue.
 - Average number of customers in the system.
 - Expected waiting time of a customer in the queue.
 - Expected waiting time of a customer in the system.
- b) A petrol pump station has three pumps. The vehicles arrive to the station according to a Poisson process at a rate of 8 per hour. The fuelling time is exponentially distributed with mean 5 minutes. Find
- Probability that a customer has to wait for service.
 - Average number of vehicles waiting in the queue.
 - Expected waiting time of a customer in the queue.
 - Expected time a customer spends at the station.
 - The traffic intensity.